arXiv:1506.08474v1 [astro-ph.CO] 28 Jun 2015

Large-scale magnetic fields, non-Gaussianity, and gravitational waves from inflation

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We explore the generation of large-scale magnetic fields in the so-called moduli inflation. The hypercharge electromagnetic fields couple to not only a scalar field but also a pseudoscalar one, so that the conformal invariance of the hypercharge electromagnetic fields can be broken. We explicitly analyze the strength of the magnetic fields on the Hubble horizon scale at the present time, the local non-Gaussianity of the curvature perturbations originating from the massive gauge fields, and the tensor-to-scalar ratio of the density perturbations. As a consequence, we find that the local non-Gaussianity and the tensor-to-scalar ratio are compatible with the recent Planck results.

Keywords: Cosmology; Particle-theory and field-theory models of the early Universe; Axions and other Nambu-Goldstone bosons; String and brane phenomenology

1. Introduction

Magnetic fields with the current strength $\sim 10^{-6}$G on 1–10kpc scale and those with 10^{-7}–10^{-6}G on 10 kpc–1Mpc scale have been detected in galaxies and clusters of galaxies, respectively. The generation mechanism of the large-scale magnetic fields in clusters of galaxies have not been understood well.[1–7]

Electromagnetic quantum fluctuations generated during inflation are considered to be the most natural origin of such large-scale magnetic fields. This is because inflation extends the coherent scale of the magnetic fields to be larger than the Hubble horizon.[8] The homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) universe is conformally flat. In addition, the electromagnetic fields are conformally invariant. Accordingly, the conformal invariance of the electromagnetic fields has to be broken at the inflationary stage. As a result, the quantum fluctuations of the electromagnetic fields can be generated.[8,9] As representative mechanisms to break the conformal invariance of the electromagnetic fields, there have

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been proposed the non-minimal coupling of the electromagnetic fields to the scalar curvature,[8,10–13] its coupling to the scalar fields,[14–18] and the trace anomaly.[19]

In this paper, we review our main results in Ref. 20. We explore a toy model of moduli inflation inspired by racetrack inflation[21] in the framework of the Type IIB string theory. The new point of our model is to take into account the coupling of the hypercharge electromagnetic fields to a scalar field as well as to a pseudoscalar one, which plays a role of the inflaton field. Only the latter coupling has been studied in the past works.[22–25] We explicitly estimate the values of current magnetic fields on the Hubble horizon scale, local non-Gaussianity of the curvature perturbations, and tensor-to-scalar ratio of density perturbations. This is the most significant consequence of the present work. We use the units $k_{\mathrm{B}} = c = \hbar = 1$ and describe the Newton's constant by $G = 1/M_{\mathrm{P}}^2$, where $M_{\mathrm{P}} = 2.43 \times 10^{18}$ GeV is the reduced Planck mass. In terms of electromagnetism, we adopt Heaviside-Lorentz units.

The paper is organized as follows. In Sec. 2, the model Lagrangian is explained, and the field equations are derived. In Sec. 3, we investigate the present magnetic field strength on the Hubble horizon scale. Furthermore, we study the non-Gaussianity of the curvature perturbations in Sec. 4, and we explore the tensor-to-scalar ratio of the density perturbations in Sec. 5. Finally, summary is given in Sec. 6.

2. Model Lagrangian

The Lagrangian is represented as

$$\begin{aligned}\mathcal{L} = & \frac{M_{\mathrm{P}}^2}{2} R - \frac{1}{4} X F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} g_{\mathrm{ps}} \frac{Y}{M} F_{\mu\nu} \tilde{F}^{\mu\nu} \\ & - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - U(\Phi) - \frac{1}{2} g^{\mu\nu} \partial_\mu Y \partial_\nu Y - V(Y) \,. \end{aligned} \tag{1}$$

Here, R is the scalar curvature, Φ is the canonical scalar field, $X \equiv \exp\left(-\lambda \Phi / M_P\right)$ with a constant λ, and $V(Y) \approx \bar{V} - (1/2)\, m^2 Y^2$, where $\bar{V}$ is a constant and m is the mass of Y. Moreover, $U(\Phi)$ and $V(Y)$ are the potentials of Φ and Y, respectively, g_{ps} is a dimensionless coupling constant, and M is a constant with the mass dimension, which corresponds to the decay constant of Y. In addition, the field strength of the $\mathrm{U}(1)_Y$ hypercharge gauge field F_μ is given by $F_{\mu\nu} = \nabla_\mu F_\nu - \nabla_\nu F_\mu$ with ∇_μ the covariant derivative, and the dual field strength of F_μ is $\tilde{F}^{\mu\nu}$. The pseudoscalar field Y corresponds to the inflaton field.

We assume the flat FLRW universe with the metric $ds^2 = -dt^2 + a^2(t) d\boldsymbol{x}^2$, where a is the scale factor. In this background, the field equations of Φ (i.e., X) and Y read $\ddot{\Phi} + 3H\dot{\Phi} + dU(\Phi)/d\Phi = 0$ and $\ddot{Y} + 3H\dot{Y} + dV(Y)/dY = 0$, respectively, where $H \equiv \dot{a}/a$ is the Hubble parameter and the dot means the derivative with respect to the cosmic time t. With the Coulomb gauge $F_0(t, \boldsymbol{x}) = 0$ and $\partial_j F^j(t, \boldsymbol{x}) = 0$, we also obtain the field equation of F_μ.

3. Magnetic field strength at the present time

For the slow-roll inflation driven by the potential of Y, the scale factor $a(t)$ can be described by $a(t) = a_k \exp\left[H_{\rm inf}\,(t - t_k)\right]$, where $a_k = a(t_k)$, t_k is the time when a comoving wavelength $2\pi/k$ of the $\mathrm{U}(1)_Y$ gauge field first crosses the horizon during inflation ($k/(a_k H_{\rm inf}) = 1$), and $H_{\rm inf}$ is the Hubble parameter during inflation. A solution of equation of motion for Y is derived as[15] $Y(t) = Y_k \exp\left[(3/2)\,\mathcal{D}H_{\rm inf}\,(t-t_k)\right]$, where $\mathcal{D} \equiv -1 \pm \sqrt{1+\left[2m/\left(3H_{\rm inf}\right)\right]}$ and $Y_k \equiv Y(t=t_k)$.

We execute the cannonical quantization of the $\mathrm{U}(1)_Y$ gauge field $F_\mu(t,\boldsymbol{x})$. We express the comoving wave number by $\boldsymbol{k}$ ($k = |\boldsymbol{k}|$). We set the x^3 axis to lie along the direction of the spatial momentum $\boldsymbol{k}$ and express the transverse directions as x^1 and x^2. By using the circular polarizations $F_\pm(k,t) \equiv F_1(k,t) \pm iF_2(k,t)$ with the Fourier modes $F_1(k,t)$ and $F_2(k,t)$ of the $\mathrm{U}(1)_Y$ gauge field, we have

$$\ddot{F}_\pm(k,t) + \left(H_{\rm inf} + \frac{\dot{X}}{X}\right)\dot{F}_\pm(k,t) + \left[1 \pm \frac{g_{\rm ps}}{M}\frac{\dot{Y}}{X}\left(\frac{k}{a}\right)^{-1}\right]\left(\frac{k}{a}\right)^2 F_\pm(k,t) = 0\,. \quad (2)$$

We numerically calculate the solution of this equation at the inflationary stage by defining the ratio $C_+(k,t) \equiv F_+(k,t)/F_+(k,t_k)$, where t_k is taken as the initial time of the numerical calculation. Here, in the short-wavelength limit of $k \to \infty$, we set $F_+(k,t) = 1/\sqrt{2kX(t)}$. Namely, we take the so-called Bunch-Davies vacuum so that in this limit the vacuum can be the one in the Minkowski space-time. As a result, without sensitive dependence of the model parameters, $C_+(k,t)$ becomes a constant after ~ 10 Hubble expansion time. after the horizon crossing during inflation.

The proper hypermagnetic field is expressed as[14] ${B_Y}_i^{\rm proper}(t,\boldsymbol{x}) = \left(1/a^2\right){B_Y}_i(t,\boldsymbol{x}) = \left(1/a^2\right)\epsilon_{ijk}\partial_j F_k(t,\boldsymbol{x})$, where ${B_Y}_i(t,\boldsymbol{x})$ is the comoving hypermagnetic field, and ϵ_{ijk} is the totally antisymmetric tensor ($\epsilon_{123} = 1$). We suppose that after inflation, the instantaneous reheating happens $t = t_{\rm R}$ (much before the electroweak phase transition (EWPT), where $T_{\rm EW} \sim 100$GeV). Carged particles are produced at the reheating stage, so that the cosmic conductivity of $\sigma_{\rm c}$ can be much larger than H. Thus, after reheating, B_Y behaves as $B_Y \propto a^{-2}$. On the other hand, the hyperelectric fields accelerats the charged particles and eventually vanish. The energy density of the magnetic fields $\rho_B(L,t)$ in the position space reads[26]

$$\rho_B(L,t) \simeq \frac{1}{8\pi^2}\frac{1}{X(t_k)}\frac{1}{\sqrt{2\xi_k}}\exp\left[2\left(\pi\xi_k - 2\sqrt{2\xi_k}\right)\right]\left(\frac{k}{a}\right)^4 \left|C_+(k,t_{\rm R})\right|^2, \quad (3)$$

with $\xi_k = \xi(t=t_k)$, where $\xi \equiv g_{\rm ps}\dot{Y}/\left(2MXH_{\rm inf}\right)$. Here, we have imposed $X(t_{\rm R}) = 1$ in order or the Maxwell theory is recovered after the instantaneous reheating ($t \geq T_{\rm R}$). We have also neglected the difference between the coefficient of the $\mathrm{U}(1)_Y$ magnetic field and that of the $\mathrm{U}(1)_{\rm em}$ one, which is $\mathcal{O}(1)$.

In Table 1, we show the cases in which $B(H_0^{-1},t_0) = \mathcal{O}(10^{-64})$ G on the Hubble horizon scale at the present time t_0 is generated for $X(t_k) = \exp\left(\chi_k\right)$ with $\chi_k = -0.940$, $g_{\rm ps} = 1.0$, $\xi_k = 2.5590616$, and $k = 2\pi/\left(2997.9h^{-1}\right)$ Mpc^{-1} with $h = 0.673$

Table 1. Magnetic field strength on the Hubble horizon scale at the present time.

	$B(H_0^{-1}, t_0)$ [G]	$B(1\text{Mpc}, t_0)$ [G]	H_{inf} [GeV]	m [GeV]	Y_k/M_{P}
(a)	7.15×10^{-64}	1.42×10^{-56}	1.0×10^{11}	2.44×10^{10}	7.70×10^{-2}
(b)	7.15×10^{-64}	1.42×10^{-56}	1.0×10^{10}	2.44×10^{9}	7.70×10^{-2}
(c)	2.33×10^{-64}	4.62×10^{-57}	1.0×10^{8}	1.0×10^{7}	1.62×10^{1}
(d)	2.33×10^{-64}	4.62×10^{-57}	1.0×10^{6}	1.0×10^{5}	1.62×10^{1}
(e)	2.85×10^{-64}	5.66×10^{-57}	1.0×10^{4}	8.0×10^{2}	2.23×10^{1}
(f)	2.85×10^{-64}	5.66×10^{-57}	1.0×10^{2}	8.0	2.23×10^{1}

(which is consistent with the recent Planck result[27]). In the cases (j) (j = a, b, c, d, e, f), we obtain $C_+(k, t_{\text{R}}) = (0.528, 0.528, 0.172, 0.172, 0.211, 0.211,)$, T_{R} [GeV] $=$ $(1.02\times10^{14}$, 3.22×10^{13}, 3.22×10^{12}, 3.22×10^{11}, 3.22×10^{10}, $3.22\times10^{9})$, and $\bar{V}/M_{\text{P}}^4 =$ $(5.07\times10^{-15}$, 5.07×10^{-17}, 5.07×10^{-21}, 5.07×10^{-25}, 5.07×10^{-29}, $5.07\times10^{-33})$. It is confirmed that $C_+(k, t_{\text{R}})$ is $\mathcal{O}(0.1)$ for the wide range of H_{inf} and m. All the results in Table 1 satisfy the constraints on the non-Gaussianity[28] and the tensor-to-scalar ratio[29] from the Planck analysis. The magnetic field strength of $\mathcal{O}(10^{-64})$ G is also compatible with both the theoretical backreaction problem[30] and the recent observational constraints from the Planck satellite.[31]

4. Non-Gaussianity of the curvature perturbations

In the context of string theories, the gauge symmetry is spontaneously broken, so that the gauge field can have the mass. Therefore, we study such a spontaneous symmetry breaking by considering the case that the $\text{U}(1)_Y$ gauge field has its coupling to other Higgs-like field φ, which can evolve to the vacuum expectation value. The kinetic term of φ reads $|D\varphi|^2$, where the covariant derivative for φ is defined by $D_\mu \equiv \partial_\mu + ig' F_\mu$ with g' the gauge coupling[24]. The gauge field acquires the mass thanks to the Higgs mechanism in terms of φ. The quantum fluctuations of φ lead to the quantum fluctuations on the mass of the gauge field. Hence, the amount of the quantum fluctuations is equal to that of quanta of the generated gauge field. Consequently, the gauge field generation yields the perturbations of number of e-folds during inflation δN. The local type non-Gaussinanity of anisotropy of the CMB radiation originates from these perturbations. The non-Gaussianity can be analyzed by exploring the curvature perturbations coming from the quantum fluctuations of φ. For the model of inflation in Ref. 25, with the COBE normalization of power spectrum of the curvature perturbations[32] $\Delta_{\mathcal{R}}^2(k) = 2.4 \times 10^{-9}$ at $k = k_* = 0.002\,\text{Mpc}^{-1}$, the local type non-Gaussianity $f_{\text{NL}}^{\text{local}}$ is given by[24] $f_{\text{NL}}^{\text{local}} \approx 1.0 \times 10^{14} \left(g'^4/\xi^6\right) \left(m^2/H_{\text{inf}}^2\right)$.

In Table 2, we list the local non-Gaussianity $f_{\text{NL}}^{\text{local}}$ of the curvature perturbations for $M = 1.0 \times 10^{-1} M_{\text{P}} = 2.43 \times 10^{17}\,\text{GeV}$, $g_{\text{ps}} = 1.0$, and $k = 2\pi/\left(2997.9h^{-1}\right)\,\text{Mpc}^{-1}$ with $h = 0.673$. Through the relation $\bar{V} = 3H_{\text{inf}}^2 M_{\text{P}}^2$, we determine the value of $\bar{V}$ as $\bar{V} = 5.07 \times 10^{-11} M_{\text{P}}^4$ for the case (A) and $\bar{V} = 5.07 \times 10^{-13} M_{\text{P}}^4$ for the case (B). The value of $C_+(k, t_{\text{R}})$ is (the case (A),

Table 2. Local type non-Gaussianity.

	$f_{\rm NL}^{\rm local}$	g'^2	$H_{\rm inf}$ [GeV]	m [GeV]	$Y_k/M_{\rm P}$
(A) @	2.70@	1.13×10^{-5}	1.0×10^{13}	2.44×10^{12}@	7.70×10^{-2}
(B) @	2.12×10^{8}@	1.0×10^{-1}	1.0×10^{12}	2.44×10^{11}@	7.70×10^{-2}

Table 3. Tensor-to-scalar ratio.

	r	$H_{\rm inf}$ [GeV]	m [GeV]	$Y_k/M_{\rm P}$
(A)	1.87×10^{-5}	1.0×10^{13}	2.44×10^{12}	7.70×10^{-2}
(B)	1.87×10^{-5}	1.0×10^{12}	2.44×10^{11}	7.70×10^{-2}

the case (B)) $= (0.528, 0.528)$. The magnetic field strength on the Hubble horizon scale at the present time $B(H_0^{-1}, t_0)$ [G] is estimated as (the case (A), the case (B)) $= (7.15 \times 10^{-64}, 7.15 \times 10^{-64})$, where we have used Eq. (3) with the absolute value of $C_+(k, t_{\rm R})$. The constraint on $f_{\rm NL}^{\rm local}$ acquired from the Planck satellite[28] is $f_{\rm NL}^{\rm local} = 2.7 \pm 5.8\,(68\%\,{\rm CL})$. It follows from Table 2 that the value of $f_{\rm NL}^{\rm local}$ in the case (A) can satisfy the constraints of the Planck results, but that of $f_{\rm NL}^{\rm local}$ in the case (B) cannot. The parameter sets whose values are close to those of the case (A) in Table 2 can also meet the constraints on $f_{\rm NL}^{\rm local}$, although the parameter space to meet the constraints of $f_{\rm NL}^{\rm local}$ is small.

5. Tensor-to-scalar ratio of the density perturbations

The definition of the tensor-to-scalar ratio r is the amplitude of tensor modes (i.e., the primordial gravitational waves) of the density perturbations divided by that of their scalar modes. With a slow-roll parameter $\epsilon \equiv \left(M_{\rm P}^2/2\right)\left[\left(dV(Y)/dY\right)/V(Y)\right]^2$, r is represented as[23] $r = 16\epsilon(t_k)$, where $\epsilon(t_k) = \epsilon(t = t_k) = 2M_{\rm P}^2 m^4 Y_k^2 / \left(2\bar{V} - m^2 Y_k\right)^2$. In Table 3, we display the values of the tensor-to-scalar ratio r in the cases (A) and (B) (which are the same in Table 2). The upper bound estimated by the Planck analysis is $r < 0.11(95\%\,{\rm CL})$.[29] It is clearly seen that this upper limit can be met in these cases.

Thus, if the magnetic fields on the Hubble horizon scale with the current field strength $\mathcal{O}(10^{-64})$ G are generated, not only the local non-Gaussianity but also the tensor-to-scalar ratio in terms of the CMB radiation, which are compatible with the recent Planck data, can be produced for a certain parameter space.

6. Summary

In the present paper, we have investigated the generation of the large-scale magnetic fields from a kind of moduli inflation. We have first analyzed the values of three cosmological observables: Current strength of the magnetic fields on the Hubble horizon scale, local non-Gaussianity, and the tensor-to-scalar ratio. We have found

that the local non-Gaussianity and tensor-to-scalar ratio obtained in this model can satisfy the recent constraints acquired from the Planck satellite.

Acknowledgments

The author would like to sincerely appreciate the significant discussions with Professor Tatsuo Kobayashi and Professor Osamu Seto and their kind meaningful suggestions and comments. Furthermore, he is really grateful to Professor Akio Sugamoto and Professor Masahiro Morikawa for their very warm advice and discussions. This work was partially supported by the JSPS Grant-in-Aid for Young Scientists (B) # 25800136 and the research-funds supplied from Fukushima University.

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